

$$\Delta\theta_{is} = [1 - sBi]^{-1/3} \left\{ 1 - \exp \left[- \frac{\left(1 - \ln \frac{\pi}{2Bi} \right)}{\pi (Bi - 1) (1 - (sBi)^{-1/3})} \frac{Pe}{p} \right] \right\}. \quad (20)$$

The values of $\Delta\theta_{is}$ calculated from Eq. (20) are shown in Fig. 1 by the dashed curves. For the practically useable region of values $h < 2$ mm, corresponding to values $sBi \equiv (m/p) \cdot Nu > 35$, Eq. (20) has an error on the order of 1%, and it can be used for practical calculations.

NOTATION

t_i and t_o , air temperatures outside the plate (for the inner and outer media, respectively); $t_o(x)$, air temperature in the slot; $t(x, y)$, temperature of the plate; $\theta_o(x)$ and $\theta(x, y)$, dimensionless temperatures; $\Delta\theta_{is}$, dimensionless extreme temperature of the plate surface; $p = \lambda/\lambda_o$, ratio of the thermal conductivities of the plate and the air; $Pe = \rho_o c_o v h / \lambda_o$, Peclet number; h , width of the slot; v , mean air velocity in the slot.

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MASS TRANSFER WITH A "MEMORY" WHEN DESCRIBING SORPTION PROCESSES

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The possibility of describing sorption processes by an integrodifferential equation of mass transfer with a "memory" is pointed out. An analytical solution of this equation is given.

When deriving the equations of sorption kinetics, the assumption that the isotherm has the form $a = f(c)$ is equivalent to the assumption that the act of adsorption is instantaneous [1, 2]. In fact, in many processes in which chemisorption plays an important part, the characteristic times of the acts of adsorption may be so great that they cannot be neglected. In order to take into account the inertia of the adsorption processes, we can consider an isotherm of the form

$$a = \gamma c + \int_0^{\tau} h(\Theta) c(\tau - \Theta) d\Theta. \quad (1)$$

In this expression the first term defines the contribution of adsorption processes with small characteristic times, while the second relates to slow adsorption processes. The general equation of the kinetics of isothermal sorption

$$\frac{\partial}{\partial \tau} (c + a) = - \operatorname{div} q_m \quad (2)$$

can be represented, using Eq. (1), in the form

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$$h(0) \frac{\partial c}{\partial \tau} + \int_0^{\tau} h'(\Theta) c(\tau - \Theta) d\Theta = -\text{div } q_m. \quad (3)$$

In this case the relaxation function $h(\Theta)$ must satisfy a number of relations:

$$h'(\Theta) \propto h(\Theta), \quad \lim_{\Theta \rightarrow 0} h(\Theta) = h(0) = 1 + \gamma, \quad \lim_{\Theta \rightarrow \infty} h(\Theta) = h(\infty) < \infty,$$

which are satisfied for an exponential function of the form

$$h(\Theta) = h(0) - [h(0) - h(\infty)] \exp(-\tau^*/\Theta). \quad (4)$$

For the class of sorption processes considered, the case of most interest is when the sorption kinetics is determined mainly by mass transfer through a boundary layer and the transition from the mobile phase into the adsorbed state. With regard to mass transfer in the mobile state, it can be assumed to be fairly intense and regarded as diffusion with a very large diffusion coefficient. The system of equations describing this process has the form

$$h(0) \frac{\partial c}{\partial \tau} + \int_0^{\infty} h'(\Theta) c(\tau - \Theta) d\Theta = -\text{div } q_m, \quad (5)$$

$$\tau = 0: \quad h(0) c(\mathbf{r}, 0) + \int_0^{\infty} h'(\Theta) c(-\Theta) d\Theta = h(\infty) c_0(\mathbf{r}),$$

$$\tau \geq 0: \quad q_m|_S = \beta [c_e(\tau) - c(\mathbf{r}, \tau)|_S].$$

To solve this problem we can use the approach employed in [3, 4]. Equations (5) can be simplified considerably if we change to mean-integral values of the concentrations

$$\langle c \rangle(\tau) = \frac{1}{V} \int_{(V)} c(\mathbf{r}, \tau) dV. \quad (6)$$

By integrating Eqs. (5) over the volume and bearing in mind that $c(\mathbf{r}, \tau)|_S = \langle c \rangle(\tau)$, we obtain

$$h(0) \frac{d\langle c \rangle}{d\tau} + \int_0^{\infty} h'(\Theta) \frac{d\langle c \rangle(\tau - \Theta)}{d\tau} d\Theta = \frac{\beta A}{V} [c_e(\tau) - \langle c \rangle(\tau)], \quad (7)$$

$$\tau = 0: \quad h(0) \langle c \rangle(\tau) + \int_0^{\infty} h'(\Theta) \langle c \rangle(-\Theta) d\Theta = h(\infty) \langle c_0 \rangle.$$

In Eqs. (7) it is best to change to the following dimensionless variables:

$$\begin{aligned} Fo &= \frac{\beta A \tau}{h(\infty) V}, \quad Fo_{\Theta} = \frac{\beta A \Theta}{h(\infty) V}, \quad Fo^* = \frac{\beta A \tau^*}{h(\infty) V}, \\ H(Fo_{\Theta}) &= \frac{h(\Theta)}{h(\infty)}, \quad H_0 = \frac{h(0)}{h(\infty)}, \quad y(Fo) = \frac{\langle c \rangle(\tau) - \langle c_0 \rangle}{\Delta c}, \\ y_e(Fo) &= \frac{c_e(\tau) - \langle c_0 \rangle}{\Delta c}. \end{aligned}$$

Equations (7) then take the form

$$H_0 \frac{dy}{dFo} + \int_0^{\infty} H'(Fo_{\Theta}) \frac{dy(Fo - Fo_{\Theta})}{dFo} dFo_{\Theta} + y = y_e, \quad (8)$$

$$Fo = 0: \quad H_0 y(0) + \int_0^{\infty} H'(Fo_{\Theta}) y(-Fo_{\Theta}) dFo_{\Theta} = 0,$$

$$H(Fo_{\Theta}) = 1 - (1 - H_0) \exp\left(-\frac{Fo^*}{Fo_{\Theta}}\right).$$

Using a Laplace transformation we can obtain the solution of Eq. (8) in the form

$$\bar{y}(p) = \frac{1}{p} \frac{p - \frac{1}{Fo^*}}{H_0 p^2 + \left(1 + \frac{1}{Fo^*}\right) p + \frac{1}{Fo^*}} \quad (9)$$

The original of (9) has the form

$$y(Fo) = 1 - \sum_{i=1}^2 \frac{1 + b_i}{1 - H_0 b_i} \exp(b_i Fo), \quad (10)$$

$$b_i = -\frac{1 + Fo^*}{2H_0} \left[1 + (-1)^i \sqrt{1 - \frac{4H_0 Fo^*}{(1 + Fo^*)^2}} \right].$$

NOTATION

c , concentration; c_0 , equilibrium concentration; c_e , concentration of the medium; S , surface of the medium; θ , variable of integration; p , Laplace variable; $h(\theta)$, relaxation function; β , mass-transfer coefficient; τ , time; Fo , dimensionless time; Fo_θ , dimensionless variable of integration; H_0 , value of the dimensionless relaxation function at the present moment; $H(Fo_\theta)$, value of the dimensionless relaxation function; y , dimensionless concentration; y_e , dimensionless concentration of the medium; Fo^* , dimensionless time of the relaxation function.

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